Design of nonuniform cosine-modulated filter-banks with the perfect-reconstruction property and arbitrary filter lengths[*]

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Abstract   In this paper, a method is proposed to construct recombination nonuniform cosine-modulated filter banks (CMFBs) with a perfect-reconstruction property and with arbitrary filter lengths. In this filter bank (FB) structure, certain channels of an original M-channel uniform FB are merged by a set of transmultiplexers (TMUXs), yielding nonuniform outputs, and the length restrictions on the original CMFB and the recombination TMUXs are avoided, allowing them to have arbitrary filter lengths. By imposing certain improved matching conditions on the prototype filters, nonuniform FBs with arbitrary filter lengths and with good properties can be obtained. As a result, the flexibility of selecting modulation types and filter lengths is greatly increased in the design of recombination nonuniform CMFBs. Another benefit due to the flexibility is that the system delay can be reduced.

Keywords: recombination nonuniform cosine-modulated filter bank, transmultiplexer, perfect-reconstruction, arbitrary-length.

The theory of M-channel uniform filter banks (FBs) with perfect-reconstruction (PR) property has been well established. Uniform filter banks have been widely used in many applications such as in communications, radar, system identification, speech coding, audio coding and image processing. However, in some applications, they are unable to provide appropriate time-frequency decomposition. For example, in approximating the time-frequency resolution of human ears, nonuniform spacing that matches the critical bands is preferred. This nonlinear arrangement is much useful when the signal energy exhibits bandwidth dependent distribution among frequency bands. Therefore, efficient structures and design procedures for general nonuniform filter banks are highly desirable. In contrast with uniform FBs, relatively few results have been reported in the study of nonuniform FBs. Structures and methods for the design of nonuniform FBs were presented in Refs. [1—6]. Direct and indirect structures of nonuniform FBs were classified in Ref. [1]. Pseudo-PR nonuniform FBs based on cosine-modulated filter banks (CMFBs) were studied in Refs. [2] and [3]. Nonuniform FBs with the direct structure were found to have a better control property with more freedom. However, there exist some cases in which the nonuniform FBs cannot be obtained by this method. Another drawback associated with this structure is that the implementation complexity is very high. More recently, another direct method for the pseudo-PR nonuniform FBs was proposed[4]. Although the sampling factors are irrational, the implementation complexity is much higher. A nonuniform FB with a two-stage structure was studied in Ref. [5]. The design method is called indirect or recombination (merging) method because of the merging operations involved. Since the analysis filters have an interpolation filter-like structure, the implementation complexity of the nonuniform FB is significantly reduced. It was shown that this indirect or recombination structure can achieve PR if both original FBs and recombination transmultiplexers (TMUXs) are PR. It was also found that if the numbers of channels of the original FBs and the recombination TMUXs are coprime, the analysis and synthesis filters can be represented as linear time invariant (LTI) systems. Due to its low design and implementation complexities, the CMFBs are used to construct original FBs and recombination TMUXs. Furthermore, since FBs and TMUXs can be designed separately provided that certain matching conditions are satisfied, the design procedures can be significantly simplified.

In this paper, a method is proposed to construct

recombination nonuniform CMFBs with PR property
and with arbitrary filter lengths, which is based on
the recombination structure shown in Ref. [5]. More
specifically, the length restrictions on the original
CMFBs and the recombination TMUXs are removed,
allowing them to have arbitrary filter lengths. By im-
posing improved matching conditions on the prototype
filters, nonuniform FBs with arbitrary filter lengths
and with good properties can be obtained. As a re-
sult, the flexibility of selecting modulation types and
filter lengths is greatly increased in the design of re-
combination nonuniform CMFBs. Another benefit
due to the flexibility is that the system delay can be
reduced to a certain degree. In addition, high stop-
band attenuation of the recombination nonuniform
FBs can still be achieved with very low design and
implementation complexities.

1 Principle of PR recombination nonuniform cosine-modulated filter banks

Fig. 1 shows the general structure of the recom-
bination nonuniform FBs. In this figure, the l-th
subband of the L-channel recombination nonuniform
FB is obtained by merging successively m_l subbands of
an M-channel uniform FB with synthesis filters of an
m_l-channel TMUX, producing a merged channel
with a decimation ratio of M/m_l. Here, r_l is the
starting index of the channels to be merged in the M-
channel uniform FB, $l = 0, 1, \cdots, L - 1$, and $i = 0,
1, \cdots, m_l - 1$. If the TMUX as shown in Fig. 1 satis-
fies the PR condition, then the merging and decom-
position operations are equivalent to a certain delay in
the m_l channels of the M-channel FB. Furthermore,
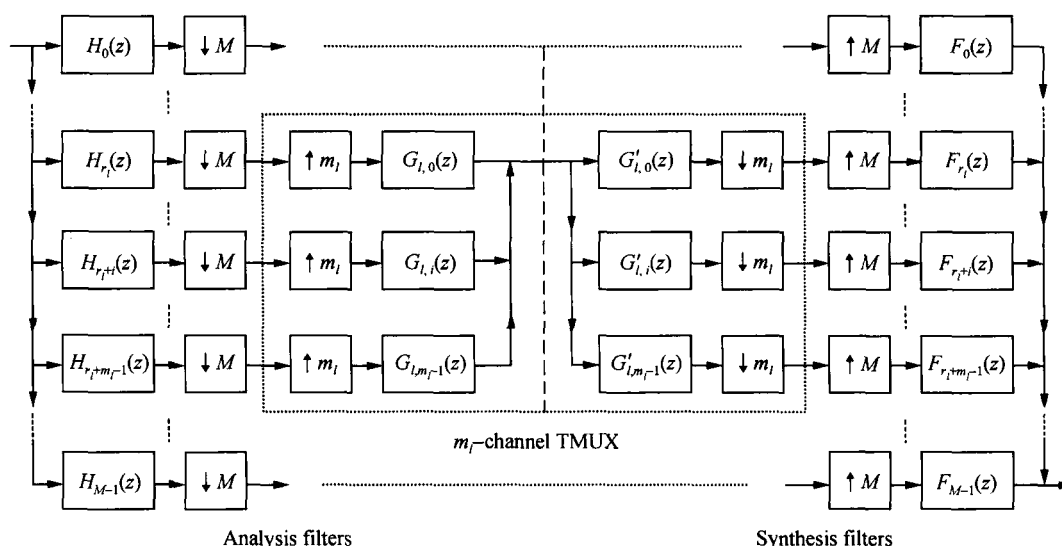if this delay is compensated in all the other branches,
the entire system is PR.

Fig. 1.   The structure of recombination nonuniform FBs.

It has been found[5] that if M and m_l are co-
prime, then the analysis and synthesis filters can be
represented by equivalent LTI systems. Due to the
good properties of CMFB, it is employed by both the
original FB and the recombination TMUX in the re-
combination nonuniform FB structure. Further, this
indirect method was extended in Ref. [6] to obtain
the nonuniform filter banks with the linear-phase
analysis and synthesis filters.

1.1 Cosine-modulated filter banks

In an M-channel uniform CMFB, the analysis
and synthesis filters, $h_k(n)$ and $f_k(n)$, are obtained
respectively by cosine-modulation of a prototype filter

$h(n)$. That is,
$$h_k(n) = h(n) \cdot c_{k,n}, \quad f_k(n) = h(n) \cdot \bar{c}_{k,n},$$
$$(1)$$
where
$$c_{k,n} = \cos\left[\frac{(2k+1)\pi}{2M}\left(n - \frac{N-1}{2}\right) + (-1)^k \frac{\pi}{4}\right],$$
$$\bar{c}_{k,n} = \cos\left[\frac{(2k+1)\pi}{2M}\left(n - \frac{N-1}{2}\right) - (-1)^k \frac{\pi}{4}\right],$$
$$k = 0, 1, \cdots, M-1, \quad n = 0, 1, \cdots, N-1. \quad (2)$$
As a general practice, the length of the prototype fil-
ter N is chosen to be $2mM$. This selection will great-
ly simplify the discussion on the PR condition. Let
$$H(z) = \sum_{q=0}^{2M-1} z^{-q} P_q(z^{2M})$$ be the type-I polyphase
decomposition of $h(n)$ and $H_k(z)$ be the z-transform

of $h_k(n)$. The PR condition can be expressed as

$$P_k(z)P_{2M-k-1}(z) + P_{M+k}(z)P_{M-k-1}(z)$$
$$= \beta \cdot z^{-\alpha},$$
$$k = 0, 1, \cdots, M - 1, \qquad (3)$$

where β is a nonzero constant and α a positive integer which is often called delay parameter. In the orthogonal CMFB, $h(n)$ is of linear-phase and hence $\alpha = m - 1$. Since $H_k(z)$ is the frequency-shifted version of the prototype filter $H(z)$, it is only necessary to minimize $H(z)$ in the stopband when the CMFB is orthogonal. In this case, $h(n)$ will be of linear-phase. The problem of designing the FBs is formulated as solving the following constrained optimization problem,

$$\min_h \Phi = \int_{\omega_s}^{\pi} | H(e^{j\omega}) |^2 d\omega \qquad (4)$$

subjected to the PR constraint in (3),

where h is a vector containing the impulse response coefficients of $h(n)$. The value of the stopband cutoff frequency ω_s depends on the desired transition bandwidth and should be between $\pi/2M$ and π/M. The solution of this nonlinear constrained optimization can be obtained using the DCONF subroutine provided by the International Mathematics and Statistics Library (IMSL).

1.2 Matching conditions

In designing the proposed CMFB-based recombination nonuniform FB, it has been found[7] that a spurious response will occur at the stopband of the resulting equivalent LTI filter if the prototype filters of the uniform FB and TMUX are not properly chosen. However, by imposing the simple matching conditions on the prototype filters, the spurious response can be effectively suppressed. The matching conditions are formulated as follows:

$$N_M/M = N_{m_l}/m_l, \qquad (5)$$

$$H(m_l\omega) = G_l(M\omega), \qquad (6)$$

where N_M and N_{m_l} are respectively the lengths of the M-channel CMFB and the m_l-channel TMUX. With the matching conditions being satisfied, the original FB and the recombination TMUX of the recombination nonuniform CMFB can be designed separately, which greatly simplifies the design procedures.

2 Arbitrary-length PR recombination nonuniform cosine-modulated filter banks

In the PR recombination nonuniform CMFBs described above, the original FBs and recombination TMUXs have the lengths of $N = 2mM$ and $N_l = 2m^{(l)}m_l$ respectively, where m and $m^{(l)}$ are certain positive integers. In this section, we will focus on the PR recombination nonuniform CMFBs with arbitrary filter lengths. Those FBs are achieved by allowing the FBs and TMUXs to have arbitrary filter lengths. The theory and design of orthogonal uniform CMFBs with the PR property and arbitrary-length have been studied in Refs. [8—10]. Nguyen and Koilpillai[8] have derived a PR condition which is exactly the same as the one with $2mM$ length. An extension based on it was made to achieve an efficient implementation[9]. In Ref. [10], a design method termed as pruning was proposed, in which an orthogonal CMFB with PR property and with arbitrary filter lengths can be generated by forcing some coefficients of a prototype filter with a length of $2mM$ being zeros. Compared with the method proposed in Refs. [8] and [9], this method is much simpler. The design for the FBs by using this method can still be considered as a $2mM$-filter design problem because the same procedure is used with the exception that certain coefficients at both ends of the prototype filter have to be zeros. Due to its simplicity, we will employ this method in the design of FBs in the recombination structure. Next, we will give a detailed description of our proposed method.

For an orthogonal CMFB with an arbitrary filter length, if the first β coefficients of the prototype filter are forced to be zeros, then the last β coefficients must be zeros as well due to the linear phase property. The length of the resulting filter is $N' = 2mM - 2\beta$, $1 \leqslant \beta \leqslant M - 1$. The modulation is the β-th sample shifted version of the original modulation in (2). This can be verified by the following equation:

$$c'_{k,n} = \cos\left[\frac{(2k+1)\pi}{2M}\left(n - \frac{N'-1}{2}\right) + (-1)^k \frac{\pi}{4}\right]$$
$$= \cos\left[\frac{(2k+1)\pi}{2M}\left(n - \frac{2mM - 2\beta - 1}{2}\right) + (-1)^k \frac{\pi}{4}\right]$$
$$= \cos\left[\frac{(2k+1)\pi}{2M}\left(n - \frac{N-1}{2} + \beta\right) + (-1)^k \frac{\pi}{4}\right]. \qquad (7)$$

Therefore, the design of $(2mM - 2\beta)$-length CMFBs is equivalent to that of $2mM$-length CMFBs. Suppose that in a recombination nonuniform FB with an arbitrary filter length, the original CMFB has a

length of $N'_M = 2mM - 2\beta_M$ and the TMUX has a length of $N'_{m_l} = 2m^{(l)}m_l - 2\beta_{m_l}$. Here, β_M and β_{m_l} are the numbers of coefficients being zeros in the prototype filters of the M-channel CMFB and m_l-channel TMUX. As mentioned earlier, in the design of recombination nonuniform FBs, the original FB and the recombination TMUX must satisfy the matching conditions of (5) and (6) to suppress the spurious response. In the following, we will examine if the same matching conditions should be satisfied in the case of $N' = 2mM - 2\beta$.

To satisfy the matching conditions, we have in the case of $N' = 2mM - 2\beta$,

$$N'_M/M = N'_{m_l}/m_l, \tag{8}$$

or equivalently

$$m - m^{(l)} = \beta_M/M - \beta_{m_l}/m_l, \tag{9}$$

with $0 < \beta_M < M$ and $0 < \beta_{m_l} < m_l$. Then we have

$$\beta_M/M = \beta_{m_l}/m_l. \tag{10}$$

For the case where M and m_l are coprime, both (10) and (8) cannot hold for integers β_M and β_{m_l}.

Instead of letting the responses of the prototype filters be in the same shape, we can make the transition bands of all the prototype filters similar by relaxing the condition that the relative weighting of the stopband and passband errors is identical. In this way, we can make all the transition bands similar with different stopband attenuation of the prototype filter by keeping $m = m^{(l)}$. Thus, the spurious responses can be suppressed significantly.

For illustration purpose, we give an example of an arbitrary-length recombination nonuniform FB with sampling factors (4/7, 3/7). The lengths of the uniform CMFB and TMUXs are respectively $N'_M = 2mM - 2\beta_M$ and $N'_{m_l} = 2m^{(l)}m_l - 2\beta_{m_l}$, with $l = 0$, 1. We set $\beta_M = 4$, $\beta_{m_0} = 2$, $\beta_{m_1} = 2$, $m = m^{(0)} = m^{(1)} = 6$. The lengths of the filters for 4-channel, 3-channel TMUXs and 7-channel CMFB are $N'_{m_0} = 2m^{(0)}m_0 - 2\beta_{m_0} = 44$, $N'_{m_1} = 2m^{(1)}m_1 - 2\beta_{m_1} = 32$ and $N'_M = 2mM - 2\beta_M = 74$ respectively. Fig. 2(a) shows the frequency responses of the equivalent LTI analysis filters and Fig. 2(b) shows those of the recombination nonuniform FB with the length of $2mM$.
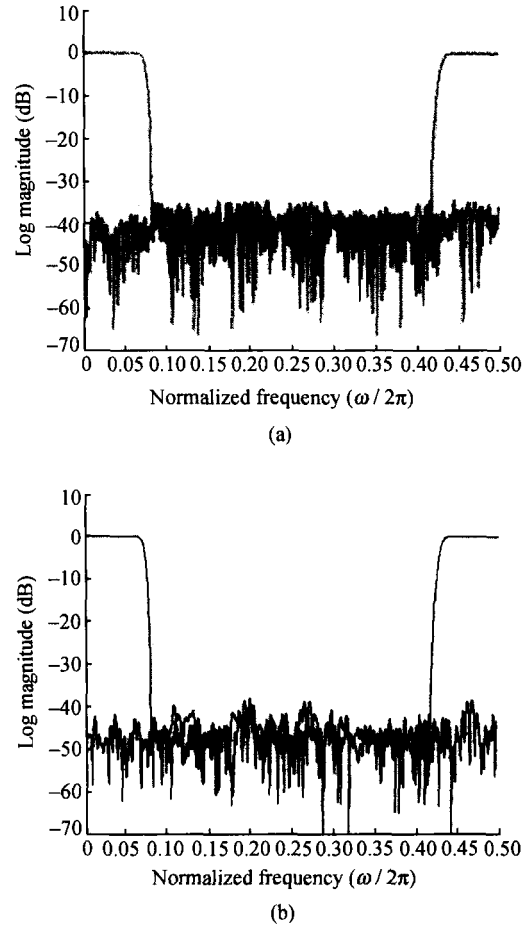

Fig. 2. Frequency responses of the equivalent LTI analysis filters of the recombination nonuniform CMFBs with sampling factors (4/7, 3/7). (a) Arbitrary-length case and (b) $2mM$ case.

3 Comparison between $2mM$-length and arbitrary-length systems

It is clear that the system delay of CMFBs with arbitrary-length of $N' = 2mM - 2\beta$ is not restricted to $2mM - 1$ samples as in the $2mM$ case. Modulations for realizing PR CMFBs with arbitrary system delays have recently been proposed in Ref. [11]. Interested readers are referred to Ref. [11] for full explanation and details. In this paper, we shall focus on the proposed pruning method for its simplicity. Since the first β coefficients of the prototype filter $h(n)$ are zero, we can shift all $h_k(n)$ and $f_k(n)$ to the left by β samples to form a new set of analysis and synthesis filters $\hat{h}_k(n) = h_k(n + \beta)$ and $\hat{f}_k(n) = f_k(n + \beta)$. Since the original system has a delay of $2mM - 1$ samples, the system with the analysis filter, $\hat{h}_k(n)$, and synthesis filter, $\hat{f}_k(n)$, will have a delay of $2mM - 2\beta - 1$ samples. This is depicted in Fig. 3. By treating the arbitrary-length CMFB as the

one with length of $N = 2mM$, we are able to preserve the form for the PR condition. This can be achieved by adding additional delays to the system, giving rise to permutation of the polyphase matrix.
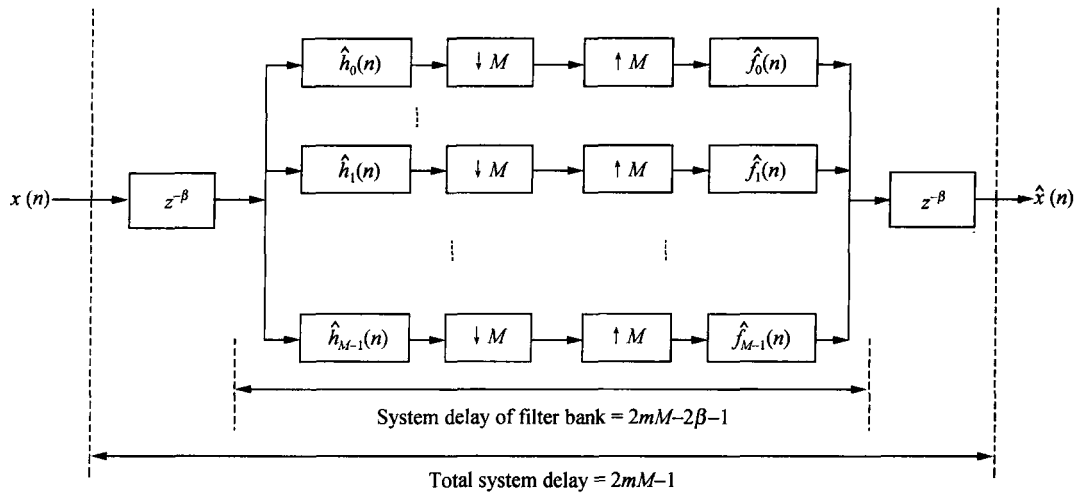

Fig. 3. System delay of an arbitrary-length CMFB.

It can be seen that the implementation of the arbitrary-length CMFB with the different system delay can be realized from an identical modulation matrix and a simple permuted structure by the use of the pruning method.

In the $2mM$ case, the overall system delay of an M-channel uniform CMFB is fixed to $2mM - 1$ samples. As mentioned above, PR TMUX can be generated from the 1-skewed PR FB, and 1-skewed PR FB $\{H_k'(z), F_k'(z)\}$ can be obtained from a standard PR FB $\{H_k(z), F_k(z)\}$[10] by choosing the filters $H_k'(z) = H_k(z)$ and $F_k'(z) = z^{-1} F_k(z)$. If a standard M-channel PR FB has a system delay of $2mM - 1$ samples, then the system delay of the corresponding 1-skewed PR FB is $2mM$ samples. Hence, the CMFB-based PR TMUX will have a delay of $2m$ samples.

In the $2mM - 2\beta$ case, the system delay of an M-channel uniform CMFB is fixed at $2mM - 2\beta - 1$ samples. If a standard PR FB has a system delay of $2mM - 2\beta - 1$ samples, then the system delay of its 1-skewed counterpart is $2mM - 2\beta$ samples. The PR TMUX, generated from the 1-skewed PR FB, still maintains a delay of $2m$ samples as well as in the $2mM$ case, because the delay of 2β samples should be compensated in this system to guarantee the delay parameter to be a multiple of M.

We now turn to the entire structure of the recombination nonuniform FB. The total system delay depends on the delay of the M-channel uniform CMFB as well as the maximum delay of the m_l-channel TMUXs.

The total system delay of a $2mM$-length recombination nonuniform CMFB is $2mM - 1 + M \cdot \max\{2m^{(l)}\}$ samples, where $2mM - 1$ and $2m^{(l)}$ are the system delay of the original M-channel CMFB and the recombination m_l-channel TMUX, respectively. While the total system delay of an arbitrary-length recombination nonuniform CMFB is $2mM - 2\beta_M - 1 + M \cdot \max\{2m^{(l)}\}$ samples, where $2mM - 2\beta_M - 1$ and $2m^{(l)}$ are the system delay of the original M-channel CMFB and the recombination m_l-channel TMUX. Fig. 4 shows the $2mM$-length and arbitrary-length recombination nonuniform FBs with their system delay. It can be seen that the system delay of the arbitrary-length recombination nonuniform FB is $2\beta_M$ samples lower than that of the $2mM$-length one.

For comparison purpose, we use the examples of the FB systems with $2mM$-length and arbitrary-length in Section 2. Table 1 gives the system delay and stopband attenuation for those two systems. We have found that using the CMFBs having arbitrary filter lengths increases the flexibility of selecting the filter length in designing recombination nonuniform FBs. In addition, by shortening the lengths of the recombination nonuniform FBs, the system delay can be reduced to a certain degree while paying a price of reducing the stopband attenuation.
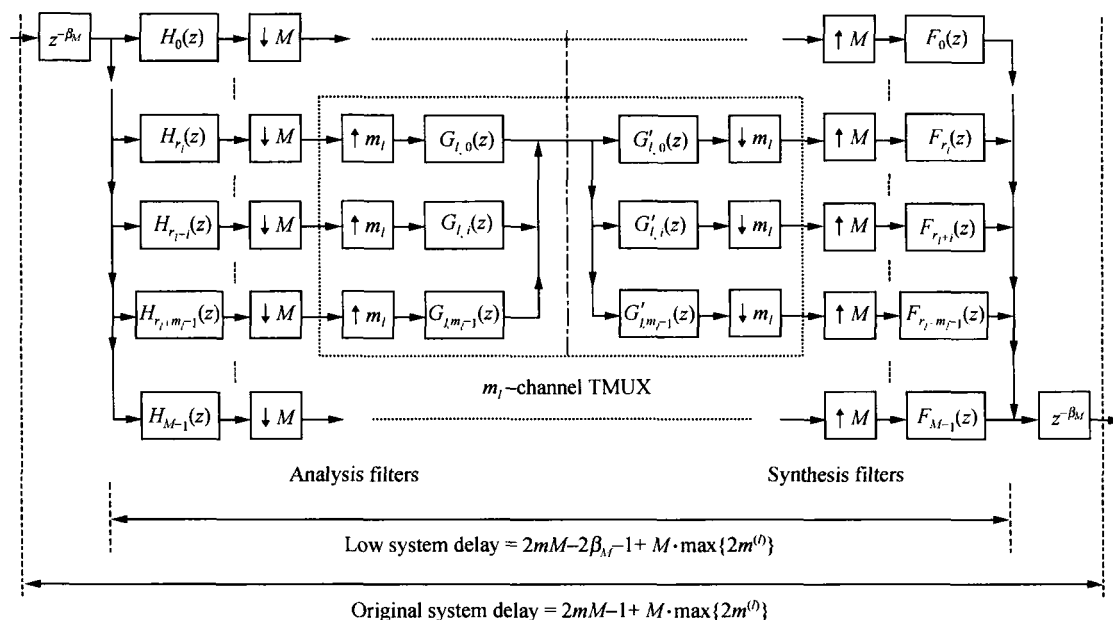
Fig. 4.   Illustration of the $2mM$-length and arbitrary-length recombination nonuniform FBs with their system delay.

Table 1.   Comparison between two types of recombination nonuniform FBs

Recombination nonuniform FB	m_l, M, β, N	System delay (sample)	Stopband attenuation (dB)
(4/7, 3/7) in $2mM$ case	$(m_0, N_{m_0}) = (4, 48)$ $(m_1, N_{m_1}) = (3, 36)$ $(M, N_M) = (7, 84)$	167	38
(4/7, 3/7) in $2mM - 2\beta$ case	$(m_0, \beta_{m_0}, N'_{m_0}) = (4, 2, 44)$ $(m_1, \beta_{m_1}, N'_{m_1}) = (3, 2, 32)$ $(M, \beta_M, N'_M) = (7, 4, 76)$	159	35

4   Conclusions

We have presented a method of designing a new class of recombination nonuniform CMFBs with PR property. The nonuniform FB is obtained by merging subbands of a uniform FB with the synthesis filters of TMUX. Both the original CMFB and the recombination TMUX can have arbitrary filter lengths, offering more flexible design of this class of recombination nonuniform FBs. As a result, the lower system delay can be obtained. In addition, high stopband attenuation of the recombination nonuniform FBs can be achieved with very low design and implementation complexities.

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